# Canonical Quantization of the Maxwell-Chern-Simons Theory in the Coulomb Gauge[*]

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## Abstract

The Maxwell-Chern-Simons theory is canonically quantized in the Coulomb gauge by using the Dirac bracket quantization procedure. The determination of the Coulomb gauge polarization vector turns out to be intrincate. A set of quantum Poincaré densities obeying the Dirac-Schwinger algebra, and, therefore, free of anomalies, is constructed. The peculiar analytical structure of the polarization vector is shown to be at the root for the existence of spin of the massive gauge quanta.The Coulomb gauge Feynman rules are used to compute the Möller scattering amplitude in the lowest order of perturbation theory. The result coincides with that obtained by using covariant Feynman rules. This proof of equivalence is, afterwards, extended to all orders of perturbation theory. The so called infrared safe photon propagator emerges as an effective propagator which allows for replacing all the terms in the interaction Hamiltonian of the Coulomb gauge by the standard field-current minimal

interaction Hamiltonian.

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# I. INTRODUCTION

As is known [1,2], the Maxwell-Chern-Simons (MCS) theory is a $(2+1)$-dimensional field model describing the coupling of charged fermions $(\bar{\psi}, \psi)$ of mass $m$ and electric charge $e$ to the electromagnetic potential $A_\mu$ via the Lagrangian density

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\theta}{4} \epsilon^{\mu\nu\alpha} F_{\mu\nu} A_\alpha + \frac{i}{2} \bar{\psi} \gamma^\mu \partial_\mu \psi - \frac{i}{2} (\partial_\mu \bar{\psi}) \gamma^\mu \psi - m \bar{\psi} \psi + e \bar{\psi} \gamma^\mu A_\mu \psi, \qquad (1.1)$$

where $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\theta$ is a parameter with dimension of mass. Neither parity nor time reversal are, separately, symmetries of the model [1]

The quantization of the MCS model in covariant gauges is free of inconsistencies [1]. On the other hand, the quantization of the free MCS theory in the Coulomb gauge already demands special care, due to the appearance of infrared divergences [1]. Although it has been argued [3] that these singularities can be avoided and some results of the Coulomb gauge version of the theory have been used from time to time for different purposes [4], the consistency of this formulation has not yet been fully established. This paper is dedicated to study the Coulomb gauge quantization of the MCS model.

In Section II we determine the polarization vector for the free MCS theory in the Coulomb gauge. This is an essential piece of information in what follows and, as will be seen, it will be done by gauge transforming the polarization vector of the Landau gauge.

In Section III the free MCS theory is canonically quantized, in the Coulomb gauge, by using the Dirac bracket quantization procedure [5–8]. After finding the equal-time commutation relations and the canonical Hamiltonian, we build the reduced phase space [6,8] and,

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[1] Throughout this paper we use natural units ($c = \hbar = 1$). Our metric is $g_{00} = -g_{11} = -g_{22} = 1$. For the $\gamma$-matrices we adopt the representation $\gamma^0 = \sigma^3, \gamma^1 = i\sigma^1, \gamma^2 = i\sigma^2$, where $\sigma^i, i = 1, 2, 3$ are the Pauli spin matrices. The fully antisymmetric tensor $\epsilon^{\mu\nu\lambda}$ is normalized such that $\epsilon^{012} = 1$ and we define $\epsilon^{ij} \equiv \epsilon^{0ij}$. Repeated greek indices sum from 0 to 2, while repeated latin indices from the middle of the alphabet sum from 1 to 2.

then, solve the Heisenberg equations of motion for the independent variables. Afterwards, the Hilbert space of physical states is constructed. As we shall see, all excitations turn out to be massive.

Section IV is dedicated to study the Poincaré symmetry for the free MCS model. We first show that there exist a set of densities with vanishing vacuum expectation values and obeying the Dirac-Schwinger equal-time commutator algebra [9]. These densities are, therefore, free of anomalies and their space integrals yield the corresponding Poincaré generators. Of particular interest is the generator of rotations because of the delicate mechanism giving rise to the spin of the gauge particle.

The fermions are brought into the game in Section V. The Dirac bracket quantization procedure is used again to quantize the full theory in the Coulomb gauge. Then, the free Hamiltonian, the interaction Hamiltonian and the Feynman rules are found. The core of the Section is concerned with the computation of the lowest order contribution to the Möller scattering amplitude by using the Coulomb gauge Feynman rules. We demonstrate that the result agrees with that found by using covariant gauge Feynman rules [10,11] and, moreover, that the infrared safe propagator [1,12] arises as an effective photon propagator which allows for replacing all non-covariant terms in the interaction Hamiltonian by the field-current minimal interaction

In Section VI we analyze the Möller scattering amplitude to all orders in perturbation theory. We prove, order by order, that the Coulomb and the covariant gauge Feynman rules yield the same result.

The conclusions are contained in Section VII.

## II. DETERMINATION OF THE POLARIZATION VECTOR IN THE COULOMB GAUGE

The dynamics of the free MCS theory is described by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\theta}{4}\epsilon^{\mu\nu\alpha}F_{\mu\nu}A_{\alpha}, \tag{2.1}$$

from which one derives the following equation of motion for the field $A^\beta$,

$$\Box A^\beta - \partial^\beta(\partial_\alpha A^\alpha) + \theta\epsilon^{\beta\mu\alpha}\partial_\mu A_\alpha = 0. \tag{2.2}$$

After taking into account the Coulomb gauge condition,

$$\chi \equiv \partial^j A^j = 0, \tag{2.3}$$

the equation of motion (2.2) reduces to

$$\Box A^\beta - \partial^\beta(\partial_0 A^0) + \theta\epsilon^{\beta\mu\alpha}\partial_\mu A_\alpha = 0. \tag{2.4}$$

In Hamiltonian language, the MCS theory possesses two first-class constraints and, hence, two subsidiary conditions are needed to fix the gauge completely [4]. Thus, one is left with only two independent variables in phase space, one coordinate and one momentum. Correspondingly, this theory only exhibits one degree of freedom in configuration space [13]. Therefore, it should be possible to write a plane wave solution of (2.4) in terms of a single polarization vector $(\epsilon^\beta(\vec{k}))$, namely,

$$A^\beta(x) = \epsilon^\beta(k)e^{ik\cdot x}. \tag{2.5}$$

When (2.5) is inserted back into (2.3) and (2.4) one obtains, respectively,

$$k^i\epsilon^i(k) = 0 \tag{2.6}$$

and

$$\Sigma^{\beta\alpha}\epsilon_\alpha(k) = 0\,, \tag{2.7}$$

where

$$\Sigma^{\beta\alpha} \equiv -k^2 g^{\beta\alpha} + k^\beta k^0 g^{0\alpha} + i\theta\epsilon^{\beta\rho\alpha}k_\rho. \tag{2.8}$$

The vanishing of the determinant of the matrix $\Sigma^{\beta\alpha}$ is a necessary and sufficient condition for the homogeneous system of equations in (2.7) to have solutions different from the trivial one $\epsilon^\beta(k) = 0$. One easily finds that

$$\det \Sigma = |\vec{k}^2| \, k^2 \, (k^2 - \theta^2). \tag{2.9}$$

Hence, there are, in principle, three independent solutions for the system (2.7). This seems to contradict the above conclusion based on the counting of the number of independent degrees of freedom. However, one is to notice that the polarization vector associated with the massless mode $k^2 = 0$ can only occur if $k^\beta = 0$ which in turn implies that $A^\beta$ is just a constant. To the same conclusion one arrives by specializing (2.7) and (2.8) to the case $\vec{k}^2 = 0$. Thus, as previously asserted, only one of the excitations is dynamical. This is the one associated with the massive mode $k^2 = \theta^2$.

We next focus on finding the polarization vector for the massive excitation $k^2 = \theta^2$. The transversality condition (2.6) is readily satisfied by choosing

$$\epsilon^i(\vec{k}) = \epsilon^{ij} \, k^j \, b(\vec{k}^2). \tag{2.10}$$

With the aim of determining the unknown function $b(\vec{k}^2)$, we replace (2.10) into (2.7). For $\beta = 0$ we obtain

$$\epsilon^0(\vec{k}) = i \, \theta \, b(\vec{k}^2), \tag{2.11}$$

while for $\beta = i$ one arrives at

$$(-k^2 + \theta^2)\epsilon^{ij} \, k^j \, b(\vec{k}^2) = 0, \tag{2.12}$$

which sais that the function $b(\vec{k}^2)$ remains unknown. One may argue that a normalization condition for the space-like vector $\epsilon^\mu(k)$, that is still lacking, is all what is needed to solve for $b(\vec{k})$. However, through this kind of device one only determines the modulus of $b(\vec{k})$, while a possible phase factor would be missed. As will be seen, it is precisely a $k$-dependent phase factor which accounts for the existence of spin in the present case. Furthermore, a simple calculation shows that, for example, $\epsilon^\mu(k)\epsilon^*_\mu(k) = -1$ leads to a function $|b(\vec{k})|$ which does not fulfills the regularity requirements assumed for a polarization vector. This difficulties recognize as common origin the fact that one is dealing with massive gauge quanta. Indeed, $\theta \neq 0$ implies in the existence of a rest frame of reference ($\vec{k} = 0$) for these particles and,

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in such frame, the Coulomb condition (2.6) becomes ambiguous. This is an entirely new situation as compared with that encountered, for instance, in the Coulomb gauge formulation of $QED_4$, where the gauge particle is massless.

We are, then, forced to adopt a new strategy for finding for $b(\vec{k}^2)$. It consists in reaching the Coulomb gauge from the Landau gauge $(\partial_\mu A_L^\mu = 0)$ through the gauge transformation linking these two gauges[2]. One is to observe that, unlike the case of the Coulomb gauge, the Landau gauge condition

$$k_\beta \epsilon_L^\beta(k) = 0 \tag{2.13}$$

remains operative even in the rest frame of reference $(\vec{k} = 0)$. In Landau gauge, (2.8) is replaced by

$$\Sigma_L^{\beta\alpha} \equiv -k^2 g^{\beta\alpha} + i\theta \epsilon^{\beta\rho\alpha} k_\rho \,, \tag{2.14}$$

whose determinant is

$$\det \Sigma_L = k^2(k^2 - \theta^2) \,. \tag{2.15}$$

As seen, there are massless and massive excitations in the Landau gauge [14].

For $k^2 = 0$, the simultaneous solving of (2.13) and

$$\Sigma_L^{\beta\alpha} \epsilon_{L,\alpha}(k) = 0 \tag{2.16}$$

is straightforward and yields

$$\epsilon_L^\beta(k) = k^\beta \eta(k) \,, \tag{2.17}$$

where $\eta(k)$ is an arbitrary function of $k$. This confirms that the massless excitations are pure gauge artefacts [15].

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[2]The gauge subscript $L$ identify quantities belonging to the Landau gauge. Coulomb gauge quantities remain without gauge identification.

On the other hand, for $k^2 = \theta^2$ one finds that

$$\epsilon_L^0(\vec{k}) = \frac{1}{|\theta|} \, \vec{k} \cdot \vec{\epsilon}_L(0), \tag{2.18a}$$

$$\epsilon_L^i(\vec{k}) = \epsilon_L^i(0) + \frac{\vec{\epsilon}_L(0) \cdot \vec{k}}{(\omega_{\vec{k}} + |\theta|) \, |\theta|} \, k^i, \tag{2.18b}$$

where $\epsilon_L^\beta(0)$ is the polarization in the rest frame of reference and $\omega_{\vec{k}} \equiv +(\vec{k}^2 + \theta^2)^{1/2}$. From Eq.(2.18) follows that [16]

$$\epsilon_L^0(0) = 0, \tag{2.19a}$$

$$\epsilon_L^2(0) = -i \, \frac{\theta}{|\theta|} \, \epsilon_L^1(0). \tag{2.19b}$$

To summarize, the physically meaningful Landau gauge polarization vector $\epsilon_L^\beta(\vec{k})$ turns out to be a complex space-like vector obeying

$$\epsilon_L^\beta(\vec{k})\epsilon_{L,\beta}(\vec{k}) = -\vec{\epsilon}_L(0) \cdot \vec{\epsilon}_L(0) = 0, \tag{2.20a}$$

$$\epsilon_L^\beta(\vec{k})\epsilon_{L,\beta}^*(\vec{k}) = -\vec{\epsilon}_L(0) \cdot \vec{\epsilon}_L^*(0) = -2 \, |\epsilon_L^1(0)|^2, \tag{2.20b}$$

where $|\epsilon_L^1(0)|$ is to be fixed by normalization.

We look next for the gauge transformation $\Lambda(x)$ linking the Landau and Coulomb gauges. It must be such that

$$\epsilon^\beta(k) = \epsilon_L^\beta(k) + i \, k^\beta \, \lambda(k), \tag{2.21}$$

where $\Lambda(x) = \lambda(k) \exp ik \cdot x$. Irrespective of whether one is dealing with the massless or with the massive mode, one finds from (2.21)

$$\lambda(k) = i \, \frac{\vec{k} \cdot \vec{\epsilon}_L(\vec{k})}{\vec{k}^2}. \tag{2.22}$$

According to (2.17), $\lambda(k)$ reduces, in the case of the massless mode, to $\lambda(k) = i\eta(k)$. When this result is replaced back into (2.21) one gets $\epsilon^\beta(k) = 0$ indicating that the massless (pure gauge) excitations are not present in the Coulomb gauge. As in 3+1-dimensions, the Coulomb gauge remains a faithful gauge in the sense of containing only physical excitations.

In the case of massive excitations one is to replace (2.18) into (2.22), thus obtaining

$$\lambda(\vec{k}) = i\frac{\vec{\epsilon}_L(0) \cdot \vec{k}}{\vec{k}^2} + i\frac{\vec{\epsilon}_L(0) \cdot \vec{k}}{(\omega_{\vec{k}} + |\theta|)\,|\theta|}. \tag{2.23}$$

From Eqs.(2.18), (2.21) and (2.23) one now finds that

$$\epsilon^0(\vec{k}) = i\,\theta\,\frac{\epsilon_L^i(0)\,\epsilon^{ij}k^j}{\vec{k}^2}, \tag{2.24a}$$

$$\epsilon^i(\vec{k}) = \left(\delta^{ij} - \frac{k^i k^j}{|\vec{k}|^2}\right)\epsilon_L^j(0) = \left[i\,\frac{\theta}{|\theta|}\,\epsilon_L^1(0)\right]\frac{\epsilon^{ij}k^j}{|\vec{k}|}e^{-i\frac{\theta}{|\theta|}\phi}, \tag{2.24b}$$

where

$$\phi \equiv \arctan\frac{k^2}{k^1} \tag{2.25}$$

is the angle between the spatial vectors $\vec{k}$ and $\vec{\epsilon}_L(0)$. In Eqs.(2.24) we quoted the final form of the components of the Coulomb gauge polarization vector. We succeded in expressing $\epsilon^\beta(\vec{k})$ in a form that makes clear that it goes continuously to the corresponding value in the rest frame of reference. The peculiar structure of $\epsilon^i(\vec{k})$ (see Eq.(2.24b)) should be observed. It is at the origin of the spin of the massive gauge quanta.

## III. CANONICAL QUANTIZATION OF THE FREE MCS THEORY IN THE COULOMB GAUGE

Within the Hamiltonian framework the free MCS theory is, as already pointed out [4], fully characterized by the canonical Hamiltonian

$$H = \int d^2z\left[\frac{1}{2}\pi_j\pi_j - \frac{\theta}{2}\pi_i\epsilon^{ik}A^k + \frac{1}{4}F^{ij}F^{ij} + \frac{\theta^2}{8}A^jA^j\right] \tag{3.1}$$

the primary first-class constraint

$$\Omega_0 \equiv \pi_0 \approx 0 \tag{3.2}$$

and the secondary firs-class constraint

$$\Omega_1 \equiv \partial^i\pi_i + \frac{\theta}{2}\epsilon^{ij}\partial^i A^j \approx 0. \tag{3.3}$$

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Here, we have designated by $\pi_\mu$ the momenta canonically conjugate to $A^\mu$. As usual, the sector $A^0, \pi_0$ can be eliminated from the phase space; $\pi_0$ is fixed by the constraint condition (3.2) while $A^0$ acts as the Lagrange multiplier of $\Omega_1$ and will be determined, after gauge fixing, as a function of the remaining canonical variables.

The quantization of the system in the Coulomb gauge, $\chi \equiv \partial^j A^j = 0$, is straightforward. The set of constraints $\Omega_1 \approx 0$ and $\Omega_2 \equiv \chi \approx 0$ is, by construction, second class and Dirac brackets [5] can be introduced in the usual manner. One then promotes the phase space variables $A^i, \pi_i$ to selfadjoint operators[3], and establishes that these operators are to obey a set of equal-time commutation rules which are abstracted from the corresponding Dirac brackets, the constraints and gauge conditions thereby translating into strong operator relations. This is the Dirac bracket quantization procedure [5–8], which presently yields

$$[A^i(x^0, \vec{x}), \, A^j(x^0, \vec{y})] = 0, \tag{3.4a}$$

$$[A^i(x^0, \vec{x}), \, \pi_j(x^0, \vec{y})] = i \, P_T^{ij}(\vec{x}) \, \delta(\vec{x} - \vec{y}), \tag{3.4b}$$

$$[\pi_i(x^0, \vec{x}), \, \pi_j(x^0, \vec{y})] = -i \, \frac{\theta}{2} \, \epsilon^{ij} \, \delta(\vec{x} - \vec{y}), \tag{3.4c}$$

where $P_T^{ij}(\vec{x}) \equiv \delta^{ij} - \partial_x^i \partial_x^j / \nabla_x^2$ and $\nabla_x^2 \equiv \partial_x^i \partial_x^i$.

We look next for the reduced phase-space, i.e., the phase-space spanned by the independent variables. It will prove convenient, for this purpose, to split $A^i(x)$ and $\pi_i(x)$ into longitudinal and transversal $(T)$ components. From $\Omega_2 \approx 0$ follows that the longitudinal component of $A^i(x)$ vanishes, whereas $\Omega_1 \approx 0$ can be used to eliminate the longitudinal component of $\pi_i(x)$ in terms of $A_T^i(x)$. As consequence, the theory can be fully phrased in terms of $A_T^i(x)$ and $\pi_i^T(x)$. It is not difficult to check that the canonical Hamiltonian (3.1) and the equal-time commutation rules (3.4), when casted in terms of the independent variables, read, respectively, as follows

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[3]To simplify the notation, we shall not distinguish between a quantum field operator and its classical counterpart

$$H = \int d^2z [\frac{1}{2}\pi_j^T \pi_j^T - \frac{\theta}{2}\pi_j^T \epsilon^{jk} A_T^k + \frac{1}{4}F^{ij}F^{ij} + \frac{\theta^2}{2}A_T^j A_T^j], \tag{3.5}$$

$$[A_T^i(x^0, \vec{x}), A_T^j(x^0, \vec{y})] = 0, \tag{3.6a}$$

$$[A_T^i(x^0, \vec{x}), \pi_j^T(x^0, \vec{y})] = i\, P_T^{ij}(\vec{x})\, \delta(\vec{x} - \vec{y}), \tag{3.6b}$$

$$[\pi_i^T(x^0, \vec{x}), \pi_j^T(x^0, \vec{y})] = 0. \tag{3.6c}$$

The Heisenberg equations of motion deriving from (3.5) and (3.6) are

$$\partial^0 A_T^i(x^0, \vec{x}) = \pi_i^T(x^0, \vec{x}), \tag{3.7a}$$

$$\partial_0 \pi_i^T(x^0, \vec{x}) = \partial^j F^{ji}(x^0, \vec{x}) - \theta^2 A_T^i(x^0, \vec{x}), \tag{3.7b}$$

which, after decoupling, yield

$$(\Box + \theta^2)A_T^i(x^0, \vec{x}) = 0, \tag{3.8}$$

showing that the excitations are all massive. As one can easily verify, the field configuration

$$A_T^{i(\pm)}(x^0, \vec{x}) = \frac{1}{2\pi}\int \frac{d^2k}{\sqrt{2\omega_{\vec{k}}}} e^{\pm i(\omega_{\vec{k}}x^0 - \vec{k}\cdot\vec{x})}\, A_T^{i(\pm)}(\vec{k}), \tag{3.9}$$

solves the equation of motion (3.8) and the equal-time commutation relations (3.6). Here, we have introduced the definitions

$$A_T^{i(\pm)}(\vec{k}) \equiv \epsilon^i(\vec{k})\, a^{\pm}(\vec{k}), \tag{3.10}$$

where $\epsilon^i(\vec{k})$ is the Coulomb gauge polarization vector, found in the previous Section, and

$$[a^-(\vec{k}), a^-(\vec{k}\prime)] = [a^+(\vec{k}), a^+(\vec{k}\prime)] = 0, \tag{3.11a}$$

$$[a^-(\vec{k}), a^+(\vec{k}\prime)] = \frac{1}{|\epsilon^1(0)|^2}\, \delta(\vec{k} - \vec{k}\prime). \tag{3.11b}$$

As known, the creation-destruction algebra (3.11) can be implemented in a Hilbert space with positive definite metric. So far, the canonical quantization of the MCS theory in the Coulomb gauge does not appear to be afflicted by ambiguities and can be sistematically implemented. In the next section we shall investigate the Poincaré symmetry of the model and the origin of the spin of the gauge particles.

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# IV. POINCARÉ INVARIANCE OF THE FREE MCS THEORY. THE SPIN OF THE MCS QUANTA

We start by considering the normally ordered composite operators

$$\Theta^{00}(x) \equiv \; : F^{0k}(x)F^{0k}(x) : \; + \frac{1}{4} : F^{\mu\nu}(x)F_{\mu\nu}(x) : , \tag{4.1a}$$

$$\Theta^{0k}(x) \equiv \; : F^{0j}(x)F^{kj}(x) : , \tag{4.1b}$$

$$\Theta^{kj}(x) \equiv \; - : F^k_\lambda(x)F^{j\lambda}(x) : \; - \frac{1}{4}\delta^{kj} : F^{\mu\nu}(x)F_{\mu\nu}(x) : . \tag{4.1c}$$

Since we are working in the Coulomb gauge, the space components of the vector $A^\mu(x)$ are purely transversal while the time component $A^0(x)$ is, as we already said, a Lagrange multiplier given in terms of the remaining variables by the expression

$$A^0(x) \; = \; \frac{\theta}{\nabla^2_x} \, \epsilon^{ij}\partial^i A^j_T(x).$$

Furthermore, all the velocities in (4.1) can be eliminated in favor of the momenta by using the Heisenberg equation of motion (3.7a). Thus, all the composite operators defined in (4.1) can be entirely written in terms of the independent phase-space variables. By using (3.6) one can check, afterwards, that these operators indeed verify the Dirac-Schwinger algebra [9],

$$[\Theta^{00}(x^0,\vec{x})\,,\,\Theta^{00}(x^0,\vec{y})] \; = \; -i\left(\Theta^{0k}(x^0,\vec{x}) \, + \, \Theta^{0k}(x^0,\vec{y})\right)\partial^x_k\delta(\vec{x}-\vec{y}), \tag{4.2a}$$

$$[\Theta^{00}(x^0,\vec{x})\,,\,\Theta^{0k}(x^0,\vec{y})] \; = \; -i\left(\Theta^{kj}(x^0,\vec{x}) \, - \, g^{kj}\,\Theta^{00}(x^0,\vec{y})\right)\partial^x_j\delta(\vec{x}-\vec{y}), \tag{4.2b}$$

$$[\Theta^{0k}(x^0,\vec{x})\,,\,\Theta^{0j}(x^0,\vec{y})] \; = \; -i\left(\Theta^{0k}(x^0,\vec{x})\partial^x_j \, + \, \Theta^{0j}(x^0,\vec{y})\partial^x_k\right)\delta(\vec{x}-\vec{y}), \tag{4.2c}$$

and can, therefore, be taken as the Poincaré densities of the free MCS theory. The generators of space-time translations ($P^\mu$), Lorentz boosts ($J^{0i}$) and spatial rotations ($J$) are defined in the standard manner,

$$P^0 \equiv \int d^2x \, \Theta^{00}(x^0,\vec{x}) \; = \; H, \tag{4.3a}$$

$$P^i \equiv \int d^2x \, \Theta^{0i}(x^0,\vec{x}), \tag{4.3b}$$

$$J^{0i} \equiv -x^0 P^i + \int d^2x \, [x^j \Theta^{00}(x^0, \vec{x})], \tag{4.3c}$$

$$J \equiv \epsilon^{ij} \int d^2x \, x^i \Theta^{0j}(x^0, \vec{x}), \tag{4.3d}$$

and can be seen to fulfill the Poincaré algebra. We stress that, within the present formulation, the commutator $[J^{0i}, J^{0k}]$,

$$[J^{0i}, J^{0k}] = -i \, \epsilon^{ik} J, \tag{4.4}$$

is free of anomalies [1].

We next analyze the spin content of the MCS quanta. By going with (3.9) into Eq.(4.1b) and with the result thus obtained into (4.3d) one finds that

$$J = i\epsilon^{jl} \int d^2k \, A_T^{m(+)}(\vec{k}) \, k^l \, \frac{\partial}{\partial k^j} \, A_T^{m(-)}(\vec{k}), \tag{4.5}$$

which after taking into account (3.10) and the explicit form of the Coulomb gauge polarization vector derived in Section II (see Eq.(2.24b)) can be casted as

$$J = |\epsilon^1(0)|^2 \frac{\theta}{|\theta|} \int d^2k \, a^+(\vec{k})a^-(\vec{k}) + i\epsilon^{jl}|\epsilon^1(0)|^2 \int d^2k \, a^+(\vec{k}) \, k^l \frac{\partial}{\partial k^j} \, a^-(\vec{k}). \tag{4.6}$$

The first term in the right hand side of (4.6) is the spin part of the total angular momentum. It originates from the exponential in (2.24b). The action of the operator $J$ on a single particle state $(a^+(\vec{k})|0>)$ can be readily derived. In particular, for the rest frame of reference one obtains

$$J\{a^+(\vec{k}=0)|0>\} = \frac{\theta}{|\theta|}\{a^+(\vec{k}=0)|0>\}, \tag{4.7}$$

which tell us that the spin of the MCS quanta is $\pm 1$ depending upon the sign of the topological mass factor [1,17].

## V. LOWEST ORDER MÖLLER SCATTERING AMPLITUDE IN THE MCS THEORY

In this Section we bring the fermions into the game. Within the Hamiltonian approach the dynamics of the MCS model in the presence of fermions is described by the canonical Hamiltonian

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$$H^F = \int d^2z \left[ \frac{1}{2}\pi_j\pi_j - \frac{\theta}{2}\pi_i\epsilon^{ik}A^k + \frac{1}{4}F^{ij}F^{ij} + \frac{\theta^2}{8}A^jA^j \right.$$
$$\left. + \frac{1}{2}\pi_\psi \cdot \gamma^0\gamma^j\partial^j\psi + \frac{1}{2}(\partial^j\pi_\psi) \cdot \gamma^0\gamma^j\psi - ie\,\pi_\psi\gamma^0\gamma^j\psi A^j - im\pi_\psi\gamma^0\psi \right], \tag{5.1}$$

the primary first-class constraint (3.2) and the secondary first constraint

$$\Omega_2^F(x) = \partial^j\pi_j(x) + \frac{\theta}{2}\epsilon^{ij}\partial^i A^j(x) - ie\pi_\psi(x) \cdot \psi(x) \approx 0. \tag{5.2}$$

Here, $\pi_\psi(x) = i\bar{\psi}(x)\gamma^0$ is the momentum canonically conjugate to the field variable $\psi(x)$ and the dot symbolizes the antisymmetrization prescription $(2A \cdot B \equiv AB - BA)$. The quantization in the Coulomb gauge is performed, as in the case of the free MCS theory, by means of the Dirac bracket quantization procedure. For the bosonic sector one gets again the commutation relations in Eq.(3.4), while the equal-time commutation relations involving the fermionic field variables are

$$[\pi_j(x^0,\vec{x})\,,\,\psi(x^0,\vec{y})] = e\left[\frac{\partial_j^x}{\nabla_x^2}\delta(\vec{x}-\vec{y})\right]\psi(x^0,\vec{y})\,, \tag{5.3a}$$

$$[\pi_j(x^0,\vec{x})\,,\,\pi_\psi(x^0,\vec{y})] = -e\left[\frac{\partial_j^x}{\nabla_x^2}\delta(\vec{x}-\vec{y})\right]\pi_\psi(x^0,\vec{y})\,, \tag{5.3b}$$

$$\{\psi(x^0,\vec{x})\,,\,\pi_\psi(x^0,\vec{y})\} = i\,\delta(\vec{x}-\vec{y}). \tag{5.3c}$$

The reduced phase-space is now spanned by the independent variables $A_T^i$, $\pi_i^T$, $\psi$ and $\pi_\psi$. In term of these variables the Hamiltonian $H^F$ splits into the sum of a free ($H_0$) plus an interacting part ($H_I$), i.e.,

$$H^F = H_0 + H_I\,, \tag{5.4}$$

where

$$H_0 = \int d^2z \left[ \frac{1}{2}\pi_j^T\pi_j^T - \frac{\theta}{2}\pi_i^T\epsilon^{ij}A_T^j + \frac{1}{4}F_T^{ij}F_T^{ij} + \frac{\theta^2}{2}A_T^jA_T^j \right.$$
$$\left. + \frac{1}{2}\pi_\psi \cdot \gamma^0\gamma^j\partial^j\psi + \frac{1}{2}(\partial^j\pi_\psi) \cdot \gamma^0\gamma^j\psi - im\pi_\psi \cdot \gamma^0\psi \right] \tag{5.5}$$

and

$$H_I = \int d^2z \left[ -ie\pi_\psi \cdot \gamma^0\gamma^j A_T^j\psi + ie\theta\pi_\psi \cdot \psi\epsilon^{ij}\frac{\partial^i}{\nabla^2}A^j + \frac{e^2}{2}(\pi_\psi \cdot \psi)\frac{1}{\nabla^2}(\pi_\psi \cdot \psi) \right]. \tag{5.6}$$

On the other hand, the only nonvanishing (anti)commutators turn out to be

$$[A_T^i(x^0, \vec{x}), \, \pi_j^T(x^0, \vec{y})] = i \, P_T^{ij}(\vec{x}) \, \delta(\vec{x} - \vec{y}), \tag{5.7a}$$

$$\{\psi(x^0, \vec{x}), \, \pi_\psi(x^0, \vec{y})\} = i \, \delta(\vec{x} - \vec{y}). \tag{5.7b}$$

The Coulomb gauge quantization of the MCS model in the presence of fermions has so far been purely formal. To test its validity, we shall next use it to compute the lowest order perturbative contribution to the electron-electron elastic scattering amplitude (Möller scattering). Since the S-matrix is a gauge invariant object, our result must coincide with that obtained for the same process when working in covariant gauges [10].

From the inspection of (5.6) follows that the contributions of order $(e^2/\theta)$ to the above mentioned amplitude, from now on referred to as $R^{(2)}$, can be grouped into four different kind of terms

$$R^{(2)} = \sum_{\alpha=1}^{4} R_\alpha^{(2)}, \tag{5.8}$$

where

$$R_1^{(2)} = -\frac{e^2}{2}(\gamma^k)_{ab}(\gamma^j)_{cd} \int d^3x \int d^3y \langle \Phi_f| T \left\{ : \bar{\psi}_a(x)\psi_b(x)A_T^k(x) : \right.$$
$$\left. \times : \bar{\psi}_c(y)\psi_d(y)A_T^j(y) : \right\} |\Phi_i\rangle, \tag{5.9a}$$

$$R_2^{(2)} = \frac{ie^2}{2}(\gamma^0)_{ab}(\gamma^0)_{cd} \int d^3x \int d^3y \delta(x^0 - y^0)G_c(\vec{x} - \vec{y})$$
$$\times \langle \Phi_f| : \bar{\psi}_a(x)\psi_b(x)\bar{\psi}_c(y)\psi_d(y) : |\Phi_i\rangle, \tag{5.9b}$$

$$R_3^{(2)} = -\frac{e^2}{2}\theta(\gamma^k)_{ab}(\gamma^0)_{cd} \int d^3x \int d^3y \langle \Phi_f| T \left\{ : \bar{\psi}_a(x)\psi_b(x)A_T^k(x) : \right.$$
$$\left. \times : \epsilon^{jm}A_T^m(y) \int d^3z \delta(y^0 - z^0)\partial_y^j G(\vec{y} - \vec{z})\bar{\psi}_c(z)\psi_d(z) : \right\} |\Phi_i\rangle, \tag{5.9c}$$

$$R_4^{(2)} = -\frac{e^2}{2}\theta^2 \epsilon^{kl} \epsilon^{jm}(\gamma^0)_{ab}(\gamma^0)_{cd}$$
$$\times \int d^3x \int d^3y \langle \Phi_f| T \left\{ : A_T^l(x) \int d^3z_1 \delta(x^0 - z^0)\partial_x^k G_c(\vec{x} - \vec{z}_1)\bar{\psi}_a(z_1)\psi_b(z_1) : \right.$$
$$\left. \times : A_T^m(y) \int d^3z_2 \delta(y^0 - z^0)\partial_y^j G_c(\vec{y} - \vec{z}_2)\bar{\psi}_c(z_2)\psi_d(z_2) : \right\} |\Phi_i\rangle. \tag{5.9d}$$

Here, $T$ is the chronological ordering operator and lower case latin letters from the beginning of the alphabet are spin indices running from 1 to 2. Also, $G_c(\vec{x} - \vec{y})$ is the Coulomb Green

function which, by definition, verifies $\nabla_x^2 G_c(\vec{x} - \vec{y}) = \delta(\vec{x} - \vec{y})$, while $|\Phi_i\rangle$ and $|\Phi_f\rangle$ denote the initial and final state of the reaction, respectively.

For the case under analysis, both $|\Phi_i\rangle$ and $|\Phi_f\rangle$ are two-electron states. Fermion states obeying the free Dirac equation in 2+1-dimensions were explicitly constructed in Ref. [10], where the notation $v^{(-)}(\vec{p})\,(\bar{v}^{(+)}(\vec{p}))$ was employed to designate the two-component spinor describing a free electron of two-momentum $\vec{p}$, energy $p^0 = +(\vec{p}^2 + m^2)^{1/2}$ and spin $s = m/|m|$ in the initial (final) state. The plane wave expansion of the free fermionic operators $\psi$ and $\bar{\psi}$ in terms of these spinors and of the corresponding creation and anhilation operators goes as usual.

In terms of the initial $(p_1, p_2)$ and final momenta $(p_1', p_2')$, the partial amplitudes in (5.9) are found to read

$$R_1^{(2)} = \frac{1}{2\pi}\delta^{(3)}(p_1' + p_2' - p_1 - p_2)$$
$$\times \left\{ [\bar{v}^{(+)}(\vec{p}_1')(ie\gamma^l)v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2')(ie\gamma^j)v^{(-)}(\vec{p}_2)]D^{lj}(k) - p_1' \longleftrightarrow p_2' \right\}, \qquad (5.10a)$$

$$R_2^{(2)} = \frac{1}{2\pi}\delta^{(3)}(p_1' + p_2' - p_1 - p_2)$$
$$\times \left\{ [\bar{v}^{(+)}(\vec{p}_1')(ie\gamma^0)v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2')(ie\gamma^0)v^{(-)}(\vec{p}_2)]\frac{i}{|\vec{k}|^2} - p_1' \longleftrightarrow p_2' \right\}, \qquad (5.10b)$$

$$R_3^{(2)} = -\frac{\theta}{2\pi}\delta^{(3)}(p_1' + p_2' - p_1 - p_2)$$
$$\times \left\{ [\bar{v}^{(+)}(\vec{p}_1')(ie\gamma^l)v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2')(ie\gamma^0)v^{(-)}(\vec{p}_2)]\Gamma^l(k) \right.$$
$$\times [\bar{v}^{(+)}(\vec{p}_1')(ie\gamma^0)v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2')(ie\gamma^l)v^{(-)}(\vec{p}_2)]\Gamma^l(-k) - p_1' \longleftrightarrow p_2' \Big\}, \qquad (5.10c)$$

$$R_4^{(2)} = \frac{\theta^2}{2\pi}\delta^{(3)}(p_1' + p_2' - p_1 - p_2)$$
$$\times \left\{ [\bar{v}^{(+)}(\vec{p}_1')(ie\gamma^0)v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2')(ie\gamma^0)v^{(-)}(\vec{p}_2)]\Lambda(k) - p_1' \longleftrightarrow p_2' \right\}, \qquad (5.10d)$$

where[4]

$$D^{lj}(k) = \frac{i}{k^2 - \theta^2 + i\epsilon}\left(\delta^{jl} - \frac{k^j k^l}{|\vec{k}|^2}\right), \qquad (5.11a)$$

_____

[4]Our convention for the Fourier integral representation is $f(x) = 1/(2\pi)^3 \int d^3k f(k) \exp(ik \cdot x)$

$$\Gamma^l(k) = \frac{\epsilon^{lj}k^j}{(k^2 - \theta^2 + i\epsilon)}\frac{1}{|\vec{k}|^2}, \tag{5.11b}$$

$$\Lambda(k) = \frac{i}{|\vec{k}|^2(k^2 - \theta^2 + i\epsilon)}, \tag{5.11c}$$

and

$$k \equiv p_1' - p_1 = -(p_2' - p_2), \tag{5.12}$$

is the momentum transfer. By going back with (5.10) into (5.8) and after taking into account (5.11) one arrives at

$$
\begin{aligned}
R^{(2)} = & \left(-\frac{e^2}{2\pi}\right)\delta^{(3)}(p_1' + p_2' - p_1 - p_2)D_{\mu\nu}(k) \\
& \times \left\{[\bar{v}^{(+)}(\vec{p}_1')\gamma^\mu v^{(-)}(\vec{p}_1)]\,[\bar{v}^{(+)}(\vec{p}_2')\gamma^\nu v^{(-)}(\vec{p}_2)] - p_1' \longleftrightarrow p_2'\right\},
\end{aligned} \tag{5.13}
$$

where

$$D_{\mu\nu}(k) = -\frac{i}{k^2 - \theta^2 + i\epsilon}\left[g_{\mu\nu} + i\theta\epsilon_{\mu\nu\rho}\frac{\bar{k}^\rho}{|\vec{k}|^2}\right] \tag{5.14}$$

and $\bar{k}^\rho = (0, \vec{k})$. In the literature [1,12], the effective Coulomb gauge propagator in Eq.(5.14) has been referred to as the infrared safe propagator. We stress that, unlike the case in electrodynamics $(\theta = 0)$, $D_{\mu\nu}(k)$ does not turn to be a covariant object. One also learns from Eq.(5.13) that, when $D_{\mu\nu}(k)$ is used as the Coulomb gauge photon propagator, one is to replace all the non-covariant terms in $H_I$ by the standard field-current minimal interaction.

As for the equivalence between the Coulomb and the covariant gauges, we start by recalling that $R^{(2)}$ is a Lorentz invariant object and, then, (5.13) can be evaluated in any Lorentz frame. In the center of mass frame, the energy transfer,

$$k^0 \equiv p_1'^0 - p_1^0 = 0,$$

vanishes and, whence, nothing changes if one replaces in the right hand side of the last mentioned equation

$$\bar{k}^\rho \longrightarrow k^\rho$$

$$|\vec{k}^2| \longrightarrow -k^2.$$

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After these replacements, Eq.(5.13) can be written

$$R^{(2)} = \left(-\frac{e^2}{2\pi}\right)\delta^{(3)}(p_1' + p_2' - p_1 - p_2)D_{\mu\nu}^L(k)$$
$$\times \left\{[\bar{v}^{(+)}(\vec{p}_1')\gamma^\mu v^{(-)}(\vec{p}_1)][\bar{v}^{(+)}(\vec{p}_2')\gamma^\nu v^{(-)}(\vec{p}_2)] - p_1' \longleftrightarrow p_2'\right\} \tag{5.15}$$

where

$$D_{\mu\nu}^L(k) = -\frac{i}{k^2 - \theta^2 + ie}\left[g_{\mu\nu} - i\theta\epsilon_{\mu\nu\rho}\frac{k^\rho}{k^2}\right] \tag{5.16}$$

is the covariant Landau gauge propagator. Again, Lorentz invariance secures that (5.15) holds in all Lorentz frames. The equivalence between the Coulomb and the Landau gauge is by now established. To extend this equivalence to other covariant gauges, it is enough to observe that $D_{\mu\nu}^L(k)$ in Eq.(5.15)is contracted into conserved currents and, hence, terms proportional $k^\mu$ can be added at will.

## VI. MÖLLER SCATTERING TO ALL ORDERS OF PERTURBATION THEORY

As it was shown in the previous Section, the Coulomb gauge Feynman rules are those of the covariant gauges with the covariant propagator replaced by the infrared safe propagator (5.14). This Section is dedicated to demonstrate that this argument alone suffices to secure that, in any order of perturbation theory, the Möller scattering amplitudes computed in the Coulomb and in the covariant Landau gauge are indeed the same. The basic observation is that the difference between the Landau and the infrared safe propagator, given at Eqs.(5.16) and (5.14), respectively, is of the form

$$D_{\mu\nu}^L(k) - D_{\mu\nu}(k) = k_\mu G_\nu(k) - k_\nu G_\mu(k), \tag{6.1}$$

where

$$G_\nu(k) \equiv \theta\frac{\epsilon_{\nu 0j}}{k^2(k^2 - \theta^2 + i\epsilon)}\frac{k^j k^0}{\vec{k}^2}. \tag{6.2}$$

Consequently, the purported proof of equivalence has now been reduced to show that if in all graphs, of a given order in perturbation theory, one of the photon propagators is replaced by the right hand side of (6.1), the sum of these modified graphs vanish.

To see how this come about we examine the generic diagram in Fig. 1, where an internal photon line has been singled out. After performing the above mentioned replacement the graph decomposes into the sum of two pieces, each containing either $k_\mu G_\nu(k)$ or $k_\nu G_\mu(k)$. We study first the part containing $k_\mu G_\nu(k)$, hereafter referred to as $P_{\mu\nu}$. We furthermore assume that the incoming and outgoing fermion lines meeting at the vertex $\mu$ carry momentum $p$ and $p + k$, respectively. By using

$$\gamma \cdot k \;=\; [\gamma \cdot (p + k) - m] - (\gamma \cdot p - m)\,, \tag{6.3}$$

one gets

$$\frac{1}{\gamma \cdot p - m} \, \gamma \cdot k \, \frac{1}{\gamma \cdot (p + k) - m} = \frac{1}{\gamma \cdot p - m} - \frac{1}{\gamma \cdot (p + k) - m}. \tag{6.4}$$

Thus, the fermion lines meeting at the vertex under analysis become amputated, one at the time. This in turn implies that $P_{\mu\nu}$ splits into the sum of two contracted subgraphs.

If the amputated line was an external fermion line, the corresponding contracted subgraph vanishes due to the on shell condition.

If the amputated line was an internal fermion line we must still distinguish two possibilities. First, the amputated line did not link directly the vertices $\mu$ and $\nu$. When this is the case, the contribution to the amplitude arising from this contracted subgraph is cancelled by the contribution made by a topollogically equivalent contracted subgraph, originating from another diagram. Secondly, the amputated line did link the vertices $\mu$ and $\nu$. Then, it is clear that the contracted subgraph contains the tadpole integral

$$\int d^3k G_\nu(k)$$

which can be seen to vanish due to symmetry requirements. Through similar arguments one shows that $P_{\nu\mu}$ also vanishes.

The proof of gauge independence for the Möller scattering amplitude in the MCS theory is now complete. It greatly helps to establish the reliability of the quantization of the MCS model in the Coulomb gauge.

## VII. CONCLUSIONS

We started in this work by looking for the Coulomb gauge polarization vector of the free MCS theory. We found impossible to determine such vector by working only within the Coulomb gauge. The reason for this being clear, the MCS theory is a gauge theory whose gauge particle is massive when formulated in the Coulomb gauge. Hence, one can think of a frame of reference where this quanta is at rest. In such frame ($\vec{k} = 0$) the Coulomb condition $\vec{k} \cdot \vec{\epsilon}(\vec{k}) = 0$ is not longer operative. The way out from the trouble consisted in finding first the polarization vectors of the Landau gauge, where there exist massive and massless gauge particles. The massless excitation is a gauge artefact that dissapears when gauge transformed into the Coulomb gauge. The polarization vector associated with the massive excitation provides, after being gauge transformed, the Coulomb gauge polarization vector, which turned out to be free of ambiguities. In 2+1-dimensions the Coulomb gauge appears to be as respectable as it is in 3+1-dimensions, in the sense that it only allows for physical excitations.

We constructed, afterwards, a set of quantum densities obeying the Dirac-Schwinger algebra. These densities were taken as the Poincaré densities and served to build the Poincaré generators. Particular attention was dedicated to the generator of spatial rotations. It was shown that the spin of the massive gauge particles originates from the highly unconventional mathematical structure of the Coulomb gauge polarization vector.

The MCS gauge particles were then allow to interact with charged fermions. A practical test of the Coulomb gauge Feynman rules for the interacting theory was carried out in connection with the Möller scattering amplitude. As demanded by gauge invariance, the result was shown to agree, to all orders in perturbation theory, with that obtained by using covariant Feynman rules. This proof was based on the observation that the Coulomb gauge Feynman rules can be phrased in terms of an effective photon propagator, the infrared safe propagator [1,12], which allows for the replacement of all terms in the interaction Hamiltonian (see Eq.(5.6)) by the standard field-current minimal interaction.

To summarize, we believe to have contributed to establish that the quantization of the MCS in the Coulomb gauge is free of inconsistencies and can be systematically carried out.

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FIGURES

FIG. 1. A generic Feynman graph contributing to the Möller scattering amplitude. An internal photon line has been singled out. The box stands for the rest of the diagram.

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